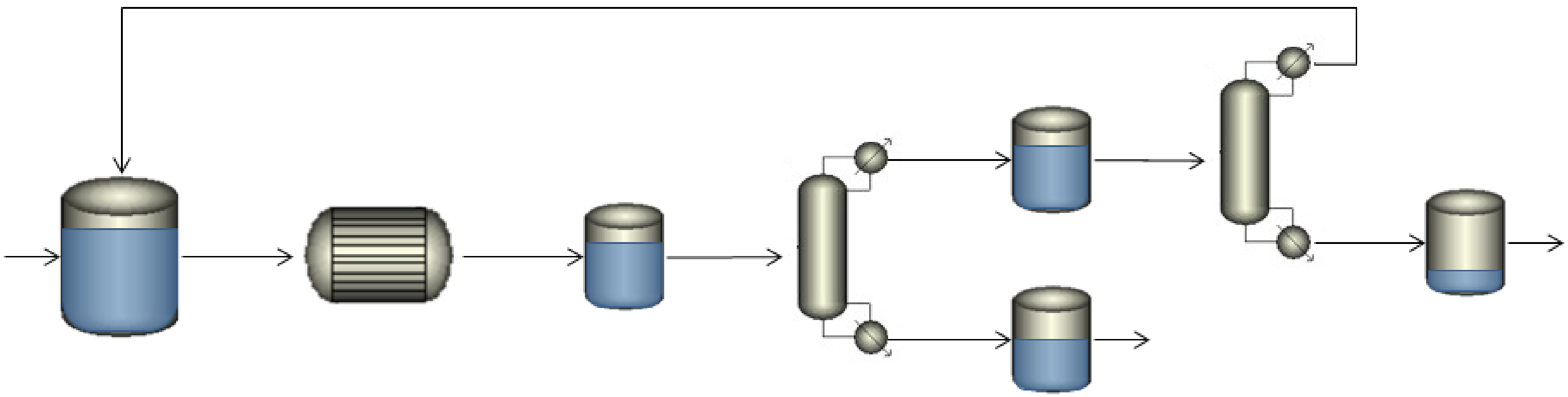
Inventory Optimization for Industrial Network Flexibility

P. Garcia-Herreros, B. Sharda, A. Agarwal, J.M. Wassick & I.E. Grossmann

Project Update
Sept 2014

Motivation

Process networks describe the operation of chemical plants

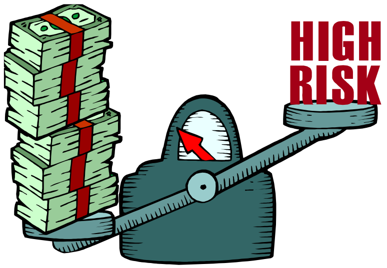


Inventories are necessary because of process uncertainty:

- Raw material storage tanks hedge against supply variability
- Intermediate storage tanks hedge against production rates variability
- Finished product inventories hedge against demand variability

Holding inventory is expensive!

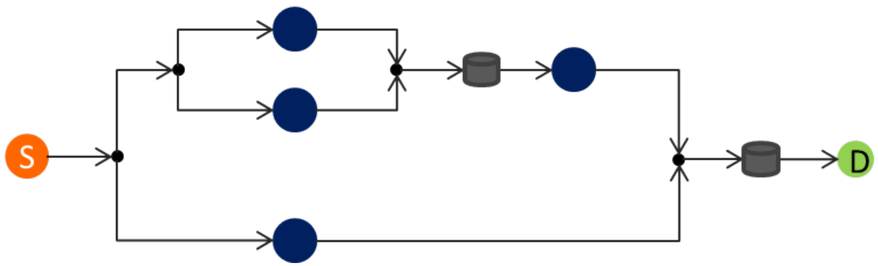
Need to trade-off between inventory and stock-out cost



Problem Statement

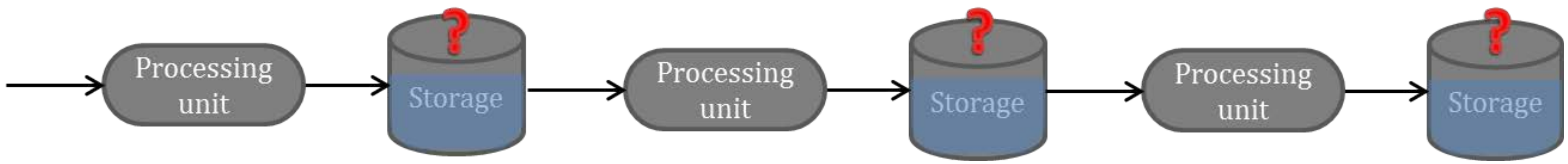
Given:

- A process network with several processing units
- A discrete time finite horizon
- Probabilistic description of supply, processing rates, and demand over the entire horizon
- Storage units



Find the operating plan that minimizes inventory and stockout costs:

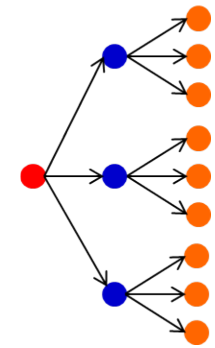
- Determine the optimal inventory management strategy in every time period
  - ➢ Location of inventory
  - ➢ Amount of inventory



Stochastic Modeling Approaches for Multiperiod Problems

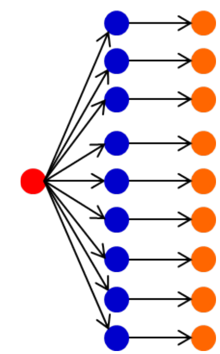
Multi-stage stochastic programming approach:

- Requires discrete uncertainty distributions
- Yields minimum expected cost
- Recourse actions are optimized for individual realizations at multiple stages
- Intractable for long time horizons (too many scenarios)



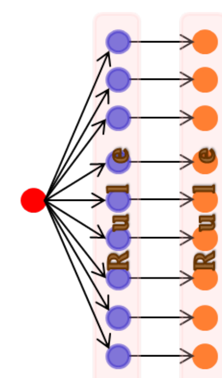
Two-stage stochastic programming approach:

- Sample paths from initial time to final time period
- Assumes that the future can be anticipated after first stage
- Recourse actions consider a single scenario
- Solvable for many scenarios



Decision rule approach:

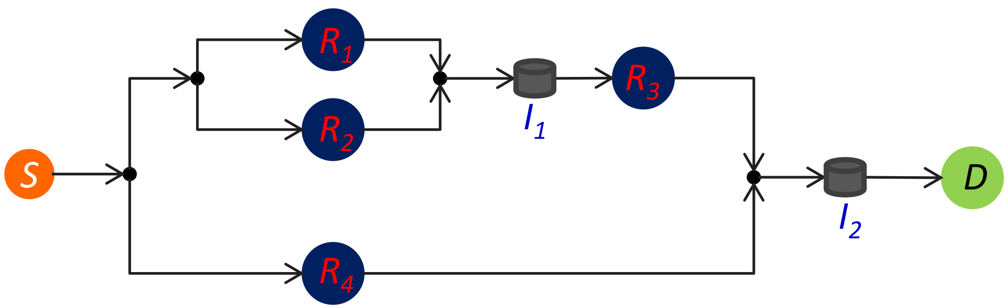
- Sample paths from initial to final time period
- Decisions in any time period are functions of the state
- Recourse actions are state functions of the individual realizations
- Does not anticipate future
- Solvable for many scenarios



Decision Rules for Inventory Management

Inventory Policies:

Rules to establish when inventories are depleted, replenished, and their priorities



Base-stock policy in capacitated network:

1. Satisfy demands (*D*) according to priorities using available supply (*S*) and production capacities (*R*)

   Update *D*, *S*, R_1, R_2, and R_3

2. Satisfy demands (*D*) according to priorities using inventories (*I*)

   Update *D*, *S*, R_1, R_2, R_3, I_1, and I_2

3. Replenish inventories according to priorities using left over supply (*S*) and production capacities (*R*)

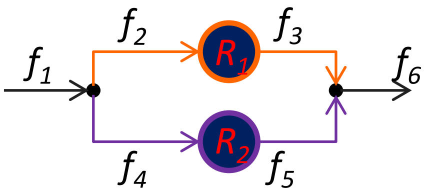
   Update *S*, R_1, R_2, R_3, I_1, and I_2

4. Stop inventory replenishments at base-stock levels (*b*)
5. Repeat for next time-period

Base-stock policy is specified by the base-stock level (*B*)

Logic of Policies

Parallel operation



Mass balances:

$$f_1 = f_2 + f_3$$
$$f_3 = C_1 f_2$$
$$\ldots$$
$$f_6 = f_3 + f_5$$

Resource capacities:

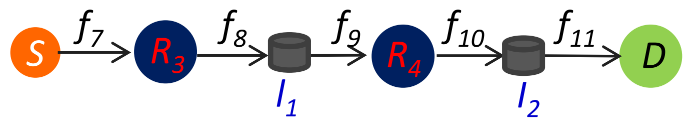
$$f_2 + u_1 = R_1$$
$$f_4 + u_2 = R_2$$

Process priorities:

Priority of R_1 over R_2

$$[u_1 = 0] \quad \vee \quad [f_4 = 0]$$

Serial operation



Mass balances:

$$f_8 = C_3 f_7$$
$$\ldots$$
$$f_{11} = C_4 f_{10}$$
$$l_1^{t+1} = l_1^t + f_8 - f_9$$

Resource capacities:

$$f_7 + u_S = S$$
$$f_7 + u_3 = R_3$$
$$f_9 + u_4 = R_4$$
$$f_{11} + so = D$$

Inventory priorities:

Priority of D over l_2 and l_2 over l_1

$$[so = 0] \quad \vee \quad [l_2 = 0]$$
$$[l_1 = 0] \quad \vee \quad [l_2 = b_2] \quad \vee \quad [u_4 = 0]$$
$$[u_S = 0] \quad \vee \quad [u_3 = 0] \quad \vee \quad [l_1 = b_1]$$

Parameters

S: Supply rate

R_i: Capacity of unit i

C_i: coefficient of unit i

D: demand rate

Variables

f_j: flowrate j

u_i: underutilization of resource i

l_k: inventory level at storage k

b_k: base-stock of storage k

so: unsatisfied demand

$f_j, u_i, l_k, b_k \geq 0$

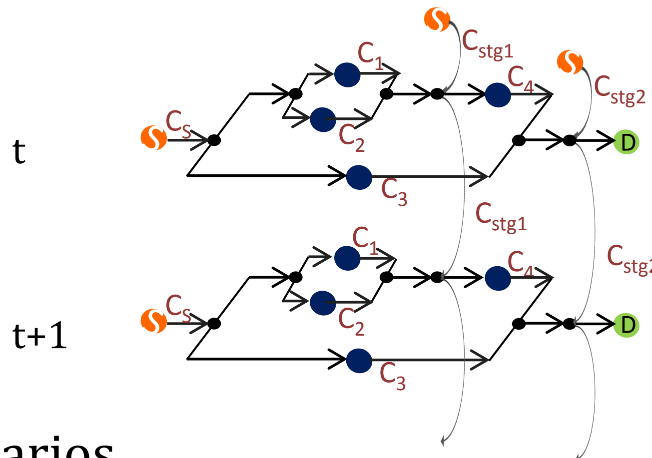
Operating plan is completely specified by the policy according to the realization of uncertain parameters (S, R_i, and D)

Multistage Formulation (MILP)

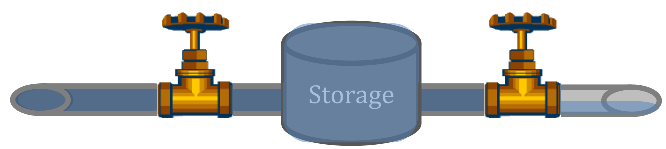
Minimize: expected inventory cost + expected stockout cost

Subject to:

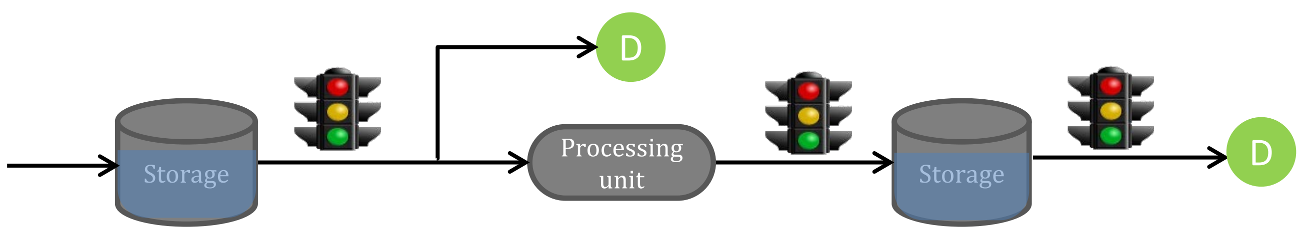
- Mass balances in all scenarios



- Capacity constraints in all scenarios



- Single policy for all scenarios in a decision stage



- Bounds

Implementation

Challenges:

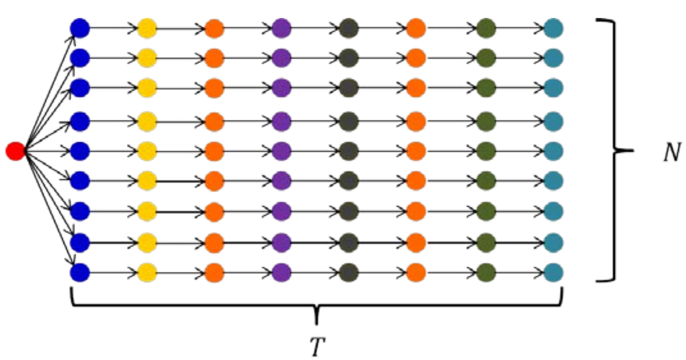
- Arbitrary distributions for uncertain parameters

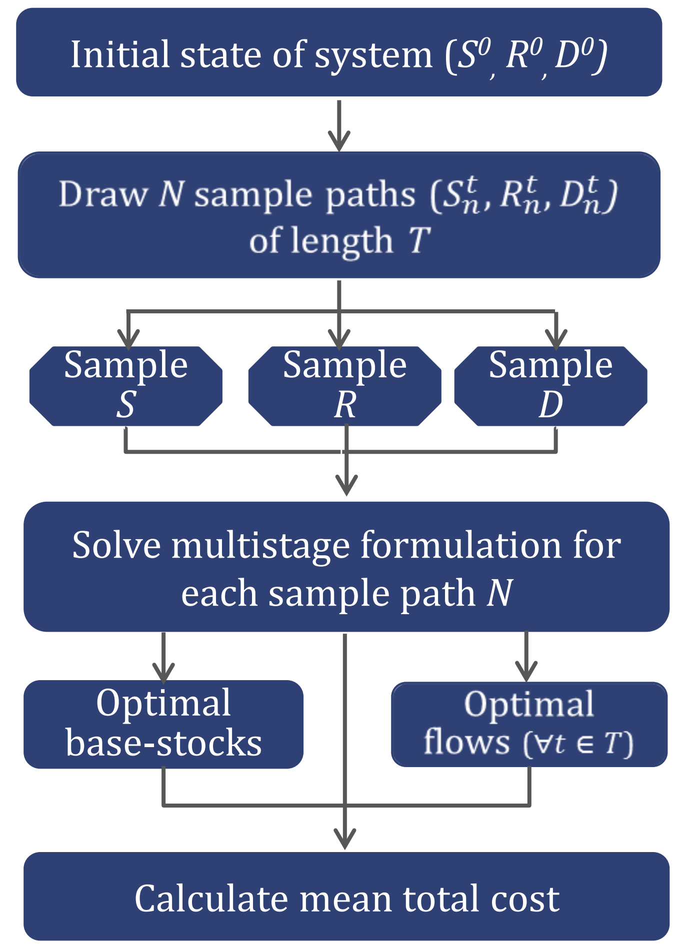
Solution approach:

- Sample-path optimization

Discrete-time samples of random parameters during planning horizon (0,T):

- Available supply: S_t
- Maximum processing rates: R_t
- Demand rates: D_t



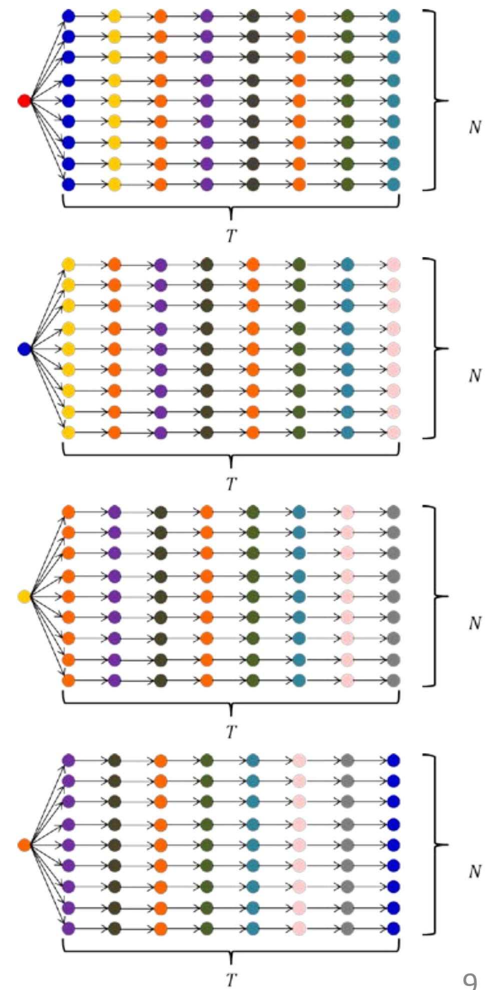


Solution approximates the optimal operating plan given the current state (at t=0) and the probabilistic model of future realizations

Evaluating the Solutions

Rolling horizon:

Simulate the implementation of the optimal first-stage decision



Algorithm:

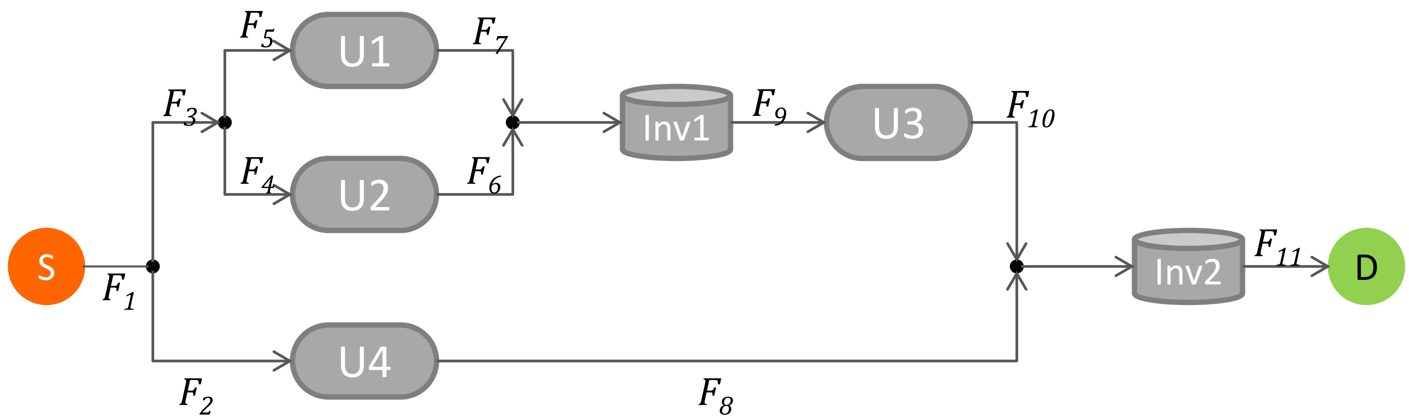
1. Initial state (S^0, R^0, D^0, $cost^0=0$)
2. Draw N sample paths of length T
3. Solve the multi-stage optimization problem
4. Accumulate the cost incurred in first stage
5. End if the end of evaluation period is reached

   Else, update initial state and return to 2

➡ Repeat the implementation of the algorithm to estimate mean and variance of the results

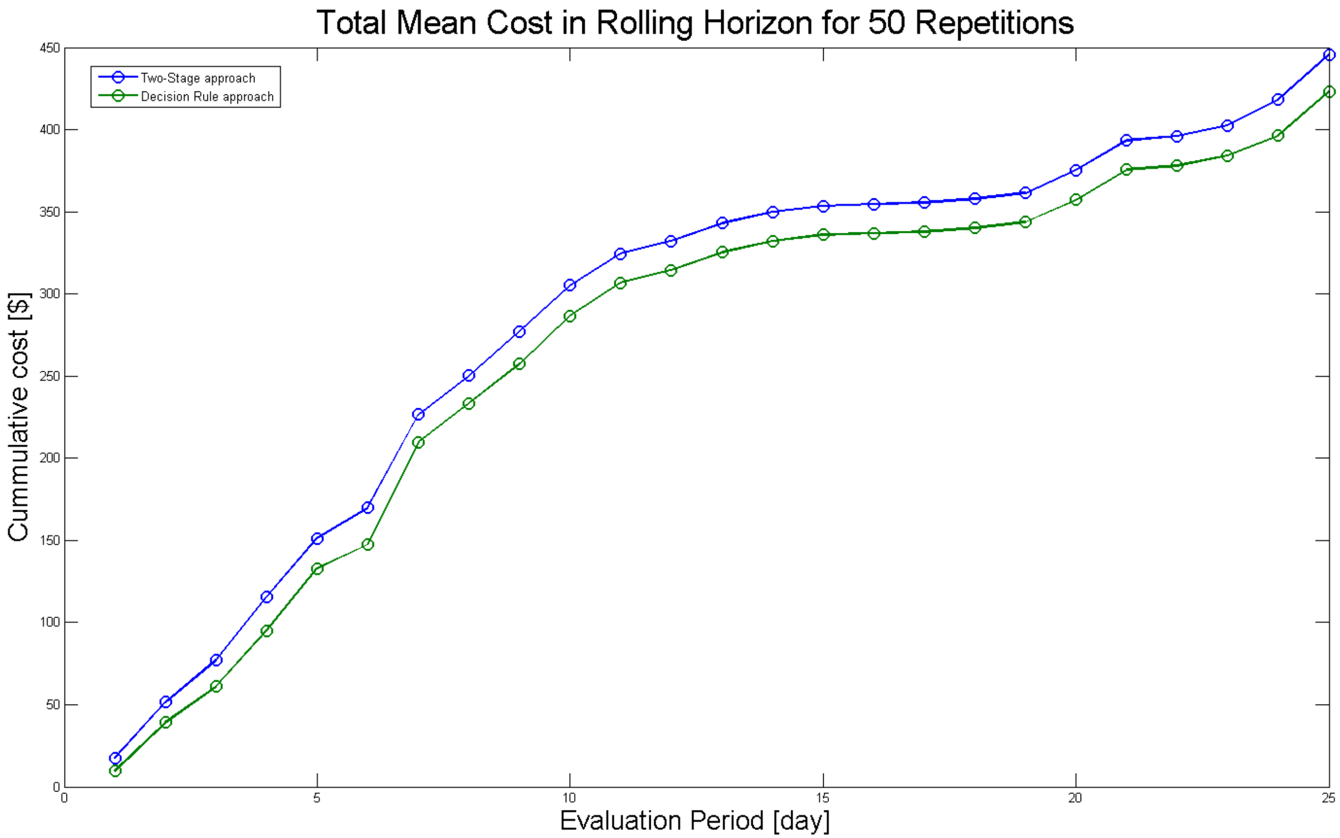
Illustrative Example

Process network with failures, uncertain supply, and uncertain demand



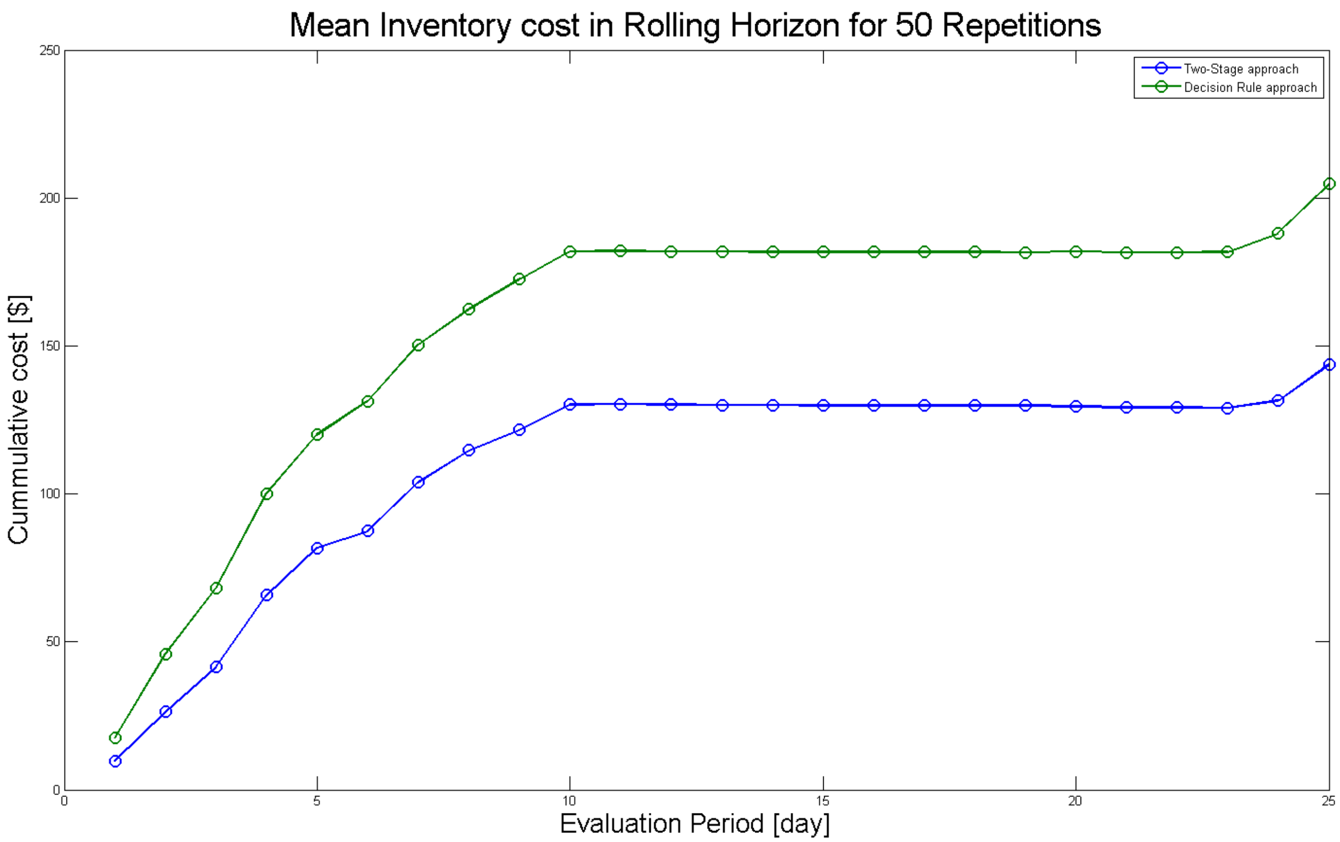
Data				
Evaluation period: H = 25 day	Number of sample paths: N = 50		Planning horizon: T = 4 day	
Supply: $S \sim N\left(12 + 2 * \sin\left(\frac{2\pi h}{H}\right), 1 + h/H\right)$ ton/day		Demand: $D \sim N\left(8 + 2 * \sin\left(\frac{2\pi h}{H}\right), 1 + h/H\right)$ ton/day		
Probability of operation:	$\pi_1 = 0.95$	$\pi_2 = 0.95$	$\pi_3 = 0.92$	$\pi_4 = 0.87$
Mass balance coefficients:	$\alpha_1 = 0.92$	$\alpha_2 = 0.90$	$\alpha_3 = 0.85$	$\alpha_4 = 0.75$
Processing capacity:	$R_1 = 5$	$R_2 = 5$	$R_3 = 7$	$R_4 = 9$
Cost coefficients:				
Production costs:	$\mu_1 = 0$	$\mu_2 = 0$	$\mu_3 = 0$	$\mu_4 = 0$
Inventory holding cost:	h_1 = 5 / (ton-day)		h_2 = 10 / (ton-day)	
Penalty for unmet demand:	p = \$50 / ton			

Results



- Two-stage approach yields higher cost when evaluated in a rolling horizon
- Expected savings are $22.1 (5%) during the 25 day horizon

Results



- Two-stage approach accumulates less inventory but has higher stockouts
- Two-stage approach assumes a degree of flexibility that is impossible to implement

Conclusions

Novelty:

- Framework for inventory planning in finite horizons
- Extension of the methodology for uncertain parameters with time-varying distributions
- Detailed logic for units in parallel and in series
- Methodology to evaluate results
- Significant improvement over two-stage approach

Impact for industrial application:

- Supply and demand forecasts can be used directly for inventory optimization
- Inventory management considering predictable and unpredictable events